# Statistics of Space Aliens

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July 7, 1998

Abstract: A model of the evolution of technological civilizations leads to the conclusion that the first civilization to appear in the galaxy would probably have plenty of time to colonize the entire galaxy before a second civilization could arise, even if there are trillions of star systems where such civilizations would eventually evolve.

## Introduction

Many people have tried to estimate the number of extraterrestrial civilizations by using the Drake Equation (Drake, 1961), which multiplies together various factors like the number of star systems, the number of planets per system, etc. Since many of the factors are entirely unknown, any kind of final answer can result. The goal in this paper is to show that even if there are a tremendous number of potential civilizations, the lucky first civilization will have a head start on the order of 100 million years before the appearance of a second. If we are that lucky first civilization, then we will have to wait about ten billion years for the next.

By "technological civilization" we mean one that has the ability and desire for interstellar travel. Such a civilization can easily colonize the entire galaxy in a few million years, an eye blink in galactic history. For supporting arguments on this, see (Zuckerman and Hart, 1995). A colonized galaxy would rather quickly have its stars surrounded by solar energy collectors, as beings made maximum use of all available energy. Such "Dyson spheres" (Dyson, 1960) would emit copious infrared radiation, and be easily recognizable from Earth with current telescopes. We do not see any Dyson spheres, so we conclude that the galaxy has not been colonized yet. Based on this, we can make some interesting calculations about how long civilizations take to develop, and how long it will be until the next one.

## The Model

The model for the appearance of civilizations consists of a uniform rate $R$ of system formation, followed by $M$ successive events, each a Poisson process of mean time $T$. The events may include the first appearance of life, the evolution of key biochemical processes, eukaryotes, multicellularity, etc. A typical model might be 32 events of mean time 100 million years each. More complicated models are possible (nonuniform system formation, different mean event times, different mean times in different systems due to different biosphere sizes, etc.), but these would greatly complicate the calculations, and would probably not contribute any fundamental insights.

**Basic parameters:**

$R$ Rate of formation of star systems suitable for civilizations.
$N$ Number of events in the evolution of a civilization.
$T$ Mean time of an event.

**Derived distributions:**

Probability density of civilization appearance time $t$ in a given system:

$$f(t)\,dt = \frac{1}{(N-1)!}\left(\frac{t}{T}\right)^{N-1} e^{-t/T}\frac{dt}{T} \tag{1}$$

Overall rate of appearance of civilizations:

$$g(t)\,dt = \int_{s=0}^{t} Rf(t-s)ds\,dt \tag{2}$$

Cumulative expected number of civilizations up to time $t$:

$$G(t) = \int_0^t g(s)\, ds \tag{3}$$

Probability density of the time of appearance of the first civilization:

$$h(t)\, dt = e^{-G(t)} g(t)\, dt \tag{4}$$

Joint probability of first civilization at $t_1$ and second at $t_2$:

$$q(t_1, t_2)\, dt_1\, dt_2 = e^{-G(t_2)} g(t_1) dt_1\, g(t_2) dt_2 \tag{5}$$

Joint probability of first civilization at $t_1$ and second at $t_1 + s$:

$$\hat{q}(t_1, s)\, dt_1\, ds = e^{-G(t_1+s)} g(t_1) g(t_1+s) dt_1 ds \tag{6}$$

Marginal distribution of the time interval $s$ between first and second civilizations:

$$p(s) ds = \int_{t_1=0}^{\infty} e^{-G(t_1+s)} g(t_1) g(t_1+s) dt_1 \tag{7}$$

**Numerical results:** The probability density distribution $p(s)$ of the interval between the first and second civilizations turns out to be approximately an exponential distribution, with the mean intervals given in Table 1. Each row represents a different rate of star system formation, in units of systems per mean event time. The columns are for different numbers of key events needed for the evolution of a civilization. As an example of using the table, suppose $10^6$ systems have formed in the $10^{10}$ year history of the galaxy, and the evolution of a civilization takes 32 key events with a mean time of 100 million years each. Then the system formation rate is $10^6$ systems per $10^2$ event times, or a rate of $10^4$. From the table, the mean time between the first two civilizations is 0.766 mean event times, or 76.6 million years.

Table 1. Mean times between 1st and 2nd civilizations
(in terms of mean event time)

| Rate, | | | | Events | | | |
|---|---|---|---|---|---|---|---|
| per event | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| 1e-03 | 999.95000 | 999.94999 | 999.94999 | 999.95014 | 999.97625 | 999.96739 | 1000.0030 |
| 1e-02 | 99.998568 | 100.00534 | 100.01646 | 100.03558 | 100.07382 | 100.14910 | 100.29638 |
| 1e-01 | 10.042342 | 10.083671 | 10.161497 | 10.306881 | 10.572211 | 11.040728 | 11.836042 |
| 1e+00 | 1.1677528 | 1.3007202 | 1.5138553 | 1.8408931 | 2.3237438 | 3.0135552 | 3.9773596 |
| 1e+01 | 0.2342562 | 0.3498452 | 0.5340489 | 0.8036302 | 1.1900764 | 1.7387205 | 2.5037201 |
| 1e+02 | 0.0626808 | 0.1289611 | 0.2547671 | 0.4696012 | 0.7819415 | 1.2297784 | 1.8655367 |
| 1e+03 | 0.0198163 | 0.0566541 | 0.1426260 | 0.3026855 | 0.5672170 | 0.9475604 | 1.4970819 |
| 1e+04 | 0.0062657 | 0.0256481 | 0.0842004 | 0.2104898 | 0.4281309 | 0.7660517 | 1.2528633 |
| 1e+05 | 0.0019790 | 0.0117674 | 0.0510466 | 0.1511569 | 0.3379296 | 0.6392454 | 1.0771975 |
| 1e+06 | 0.0006261 | 0.0054301 | 0.0314289 | 0.1108497 | 0.2726252 | 0.5405007 | 0.9438092 |
| 1e+07 | 0.0001980 | 0.0025150 | 0.0195294 | 0.0824666 | 0.2234091 | 0.4659647 | 0.8385665 |
| 1e+08 | 0.0000626 | 0.0011660 | 0.0122016 | 0.0619768 | 0.1852362 | 0.4067610 | 0.7532391 |
| 1e+09 | 0.0000197 | 0.0005409 | 0.0076559 | 0.0469203 | 0.1549792 | 0.3582943 | 0.6829621 |
| 1e+10 | 0.0000062 | 0.0002510 | 0.0048126 | 0.0357124 | 0.1305905 | 0.3179054 | 0.6231450 |
| 1e+11 | 0.0000019 | 0.0001164 | 0.0030295 | 0.0272888 | 0.1106688 | 0.2837601 | 0.5700860 |
| 1e+12 | 0.0000006 | 0.0000540 | 0.0019086 | 0.0209117 | 0.0942212 | 0.2545491 | 0.5254267 |

## Bayesian Distribution of Mean Evolution Times

It is often assumed that our time of evolution is typical, by the Copernican principle. But if in fact we are the very lucky first civilization, then by definition we are not typical, and we cannot infer much about the rate of evolution elsewhere. Certainly evolutionary theory has not progressed to the point of predicting the rate of evolution of technological civilizations! The typical time for evolution may be trillions of years, for all we know. But some interesting calculations can still be done.

Here we use Bayesian analysis to get the posterior probability distribution of the mean time for the evolution of civilization (from system formation), given that we are first. The inputs to this model are the observed time of out solar system formation, our observed evolution time, an assumed number of key evolution events, and an assumed rate of system formation.

Further notation:

| | |
|---|---|
| $t_1$ | Time of solar system formation, i.e. 6 billion years. |
| $t_2$ | Time of our evolution, i.e. 4 billion years. |
| $\alpha(t_1, t_2\|T)dt_1 dt_2$ | Conditional joint distribution. |
| $\beta(T\|t_1, t_2)dT$ | Posterior distribution of $T$. |
| $\rho(T)dT$ | Prior distribution of $T$. |

Bayes' Theorem, continous version:

$$\beta(T|t_1, t_2)dT = \frac{\alpha(t_1, t_2|T)dt_1 dt_2 \rho(T)dT}{\int_{T=0}^{\infty} \alpha(t_1, t_2|T)dt_1 dt_2 \rho(T)dT} \tag{8}$$

Conditional joint distribution for first civilization having system formation at $t_1$ and evolution time $t_2$:

$$\alpha(t_1, t_2|T)dt_1 dt_2 = e^{-G(t_1+t_2)} R\, dt_1 f(t_2) dt_2 \tag{9}$$

Canonical ignorant prior distribution (an improper distribution, but suitable when all you know about a random variable is that it is positive) :

$$\rho(T)dT = \frac{dT}{T} \tag{10}$$

Net result:

$$\beta(T|t_1, t_2) = \frac{e^{-G(t_1+t_2)} R\, f(t_2)/T}{\int_{T=0}^{\infty} e^{-G(t_1+t_2)} R\, f(t_2) \frac{1}{T} dT} \tag{11}$$

**Numerical results:** Table 2 gives the median total evolution time based on the median of the distribution for $\beta$. This particular table is calculated for parameters suitable for our solar system, $t_1 = 6 \times 10^9$ years for the creation of our solar system, and $t_2 = 4 \times 10^9$ years for the evolution of ourselves. Note that the system formation rate is per year, not per mean event time. The table divides into two regimes. At low rates of system formation, that formation is the determining factor in the rate of civilizations, and our time of evolution is typical. We just happened to be very lucky that our system formed early, or at all. On the other hand, at high rates of system formation, evolution becomes the determining factor, and typical evolution times are much longer than ours. We just got very lucky in key events happening quickly.

Table 2. Bayesian median evolution times, years
Solar system formed at 6e+09 yr, time for our evolution 4e+09 yr.

| Rate, | Events | | | | | | |
|---|---|---|---|---|---|---|---|
| per year | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| 1e-12 | 5.79e+09 | 4.78e+09 | 4.36e+09 | 4.18e+09 | 4.09e+09 | 4.04e+09 | 4.02e+09 |
| 1e-11 | 5.96e+09 | 4.86e+09 | 4.41e+09 | 4.20e+09 | 4.10e+09 | 4.05e+09 | 4.03e+09 |
| 1e-10 | 8.07e+09 | 5.82e+09 | 4.88e+09 | 4.43e+09 | 4.20e+09 | 4.10e+09 | 4.05e+09 |
| 1e-09 | 6.99e+10 | 2.26e+10 | 1.28e+10 | 8.94e+09 | 6.32e+09 | 4.79e+09 | 4.32e+09 |
| 1e-08 | 7.26e+11 | 9.33e+10 | 3.35e+10 | 1.91e+10 | 1.40e+10 | 1.16e+10 | 1.03e+10 |
| 1e-07 | 7.21e+12 | 3.05e+11 | 6.67e+10 | 3.02e+10 | 1.93e+10 | 1.48e+10 | 1.25e+10 |
| 1e-06 | 7.20e+13 | 9.76e+11 | 1.25e+11 | 4.41e+10 | 2.47e+10 | 1.77e+10 | 1.45e+10 |
| 1e-05 | 7.20e+14 | 3.10e+12 | 2.28e+11 | 6.21e+10 | 3.08e+10 | 2.03e+10 | 1.61e+10 |
| 1e-04 | 7.20e+15 | 9.80e+12 | 4.11e+11 | 8.62e+10 | 3.75e+10 | 2.33e+10 | 1.77e+10 |
| 1e-03 | 7.20e+16 | 3.10e+13 | 7.38e+11 | 1.18e+11 | 4.52e+10 | 2.59e+10 | 1.88e+10 |
| 1e-02 | 7.20e+17 | 9.80e+13 | 1.32e+12 | 1.60e+11 | 5.40e+10 | 2.88e+10 | 1.97e+10 |
| 1e-01 | 7.20e+18 | 3.10e+14 | 2.35e+12 | 2.17e+11 | 6.38e+10 | 3.21e+10 | 2.15e+10 |
| 1e+00 | 7.20e+19 | 9.80e+14 | 4.19e+12 | 2.93e+11 | 7.53e+10 | 3.56e+10 | 2.36e+10 |
| 1e+01 | 7.20e+20 | 3.10e+15 | 7.45e+12 | 3.93e+11 | 8.89e+10 | 3.93e+10 | 2.38e+10 |
| 1e+02 | 7.20e+21 | 9.80e+15 | 1.33e+13 | 5.28e+11 | 1.04e+11 | 4.32e+10 | 2.60e+10 |
| 1e+03 | 7.20e+22 | 3.10e+16 | 2.36e+13 | 7.06e+11 | 1.21e+11 | 4.71e+10 | 2.82e+10 |

## For how long do we have the galaxy to ourselves?

Given that we are the first civilization in the galaxy, how long can we expect to wait until the appearance of the second? This can be found by using the Bayes posterior distribution of event times found in the previous section to calculate the distribution of the time of appearance of the next civilization. Writing the mean event time $T$ explicitly in the probability formulas of the first section, we have for the density function for the appearance of civilizations

$$g(t,T)\,dt = \int_{s=0}^{t} Rf(t-s,T)ds\,dt \tag{12}$$

Cumulative expected number of civilizations:

$$G(t,T) = \int_{t_1+t_2}^{t} g(s,T)\,ds \tag{13}$$

Probability density of first civilization:

$$h(t,T)\,dt = e^{-G(t,T)}g(t,T)\,dt \tag{14}$$

Overall mean time until next civilization:

$$\mu = \int_{0}^{\infty}\int_{t_1+t_2}^{\infty} t\,e^{-G(t,T)}g(t,T)\,dt\beta(T|t_1,t_2)dT \tag{15}$$

**Numerical results.** Given that we are first, and given our system formation at $6\times10^9$ years, and our evolution time of $4\times10^9$ years, Table 3 presents the expected mean time until the next civilization. Note that the system formation rate is per year, not per mean event time. For low system formation rates, we basically just have to wait a long time until the next system forms; evolution of civilization happens quickly after that. For higher system formation rates, we just have to wait until somebody is as lucky as we were, which takes about the same amount of time. In any case, the mean waiting time is at least 10 billion years.

This is much longer than the mean interval between civilizations in Table 1 since that calculation did not take as given the time of appearance of the first civilization.

Table 3. Mean times until the next civilization, years

| Rate, per year | Events 1 | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|---|
| 1e-12 | 1.04e+12 | 1.03e+12 | 1.02e+12 | 1.02e+12 | 1.02e+12 | 1.02e+12 | 1.02e+12 |
| 1e-11 | 1.24e+11 | 1.14e+11 | 1.12e+11 | 1.11e+11 | 1.11e+11 | 1.11e+11 | 1.11e+11 |
| 1e-10 | 2.71e+10 | 2.25e+10 | 2.08e+10 | 2.02e+10 | 2.01e+10 | 2.01e+10 | 2.01e+10 |
| 1e-09 | 2.00e+10 | 1.50e+10 | 1.28e+10 | 1.16e+10 | 1.11e+10 | 1.10e+10 | 1.10e+10 |
| 1e-08 | 2.02e+10 | 1.51e+10 | 1.26e+10 | 1.14e+10 | 1.08e+10 | 1.04e+10 | 1.02e+10 |
| 1e-07 | 2.01e+10 | 1.51e+10 | 1.26e+10 | 1.14e+10 | 1.07e+10 | 1.04e+10 | 1.02e+10 |
| 1e-06 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.04e+10 | 1.03e+10 |
| 1e-05 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.04e+10 | 1.03e+10 |
| 1e-04 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.04e+10 | 1.03e+10 |
| 1e-03 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.03e+10 | 1.03e+10 |
| 1e-02 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.03e+10 | 1.01e+10 |
| 1e-01 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.03e+10 | 1.02e+10 |
| 1e+00 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.06e+10 | 1.04e+10 | 1.03e+10 |
| 1e+01 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.07e+10 | 1.04e+10 | 1.01e+10 |
| 1e+02 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.06e+10 | 1.04e+10 | 1.02e+10 |
| 1e+03 | 2.01e+10 | 1.50e+10 | 1.25e+10 | 1.13e+10 | 1.06e+10 | 1.03e+10 | 1.04e+10 |

## Conclusion

The arguments presented here have been in terms of our own galaxy. But they equally apply to other galaxies, and indeed the entire visible universe. There is no sign of other galaxies being colonized, i.e. no infrared radiation from Dyson spheres, so the logical conclusion is that we are the first technological civilization in the visible universe, and it will be a long time before there is another! Sorry, X-Files fans.

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